Gamow on Newton: Another Look at Centripetal Acceleration

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Presented here is an adaptation of George Gamow's derivation of the centripetal acceleration formula as it applies to Earth's orbiting Moon. The derivation appears in Gamow's short but engaging book *Gravity,*[1] first published in 1962, and is essentially a distillation of Newton's work. While *TPT* contributors have offered several insightful derivations of the formula,[2,3] students might find this "real world" version more accessible. In addition, this particular derivation requires a durable understanding of both the Galilean equations and Newtonian laws of motion, and therefore serves as review of these key concepts.

We begin by asking what would happen to the Moon's motion if the Earth were to suddenly vanish. As shown in Fig. 1, it would continue off in a straight line in accordance with the first law. Equating the three sides of the newly formed right triangle using the Pythagorean theorem, we get:

$$(R+x)^2 = R^2 + d^2 \quad . \tag{1}$$

In the limit of shorter and shorter time intervals, distance d and arc length s approach one another and we can therefore equate d to the product of the Moon's tangential velocity and time interval (Δt). Making this substitution and multiplying out the square, we get:

$$R^2 + 2Rx + x^2 = R^2 + (v\Delta t)^2 \, . \tag{2}$$

Finally, the R^2 is cancelled from both sides and the entire equation is divided by $2R$:

$$x + \frac{x^2}{2R} = \frac{v^2 \Delta t^2}{2R}. \tag{3}$$

"Now," says Gamow, "comes an important argument." When we consider shorter and shorter time intervals, both terms on the left side of Eq. (3) approach zero. Rewriting the second term as $\frac{x(x)}{2R}$ and realizing that $\frac{x}{2R}$ is much less than one, this term becomes vanishingly small and can therefore be ignored. Gamow argues this point by noting that since the second term contains the square of x, it "goes to zero *faster* than the first and we can write":

$$x = \frac{1}{2}\left(\frac{v^2}{R}\right)\Delta t^2. \tag{4}$$

Gamow asks us to view length x as "the distance [the Moon] has fallen toward the Earth during the time interval Δt." He continues, "In discussing Galileo's studies of the law of fall, we have seen that the distance traveled during the time interval Δt is $\frac{1}{2}(a)\Delta t^2$, where a is the acceleration, so that, comparing the two expressions we conclude that $\frac{v^2}{R}$ represents

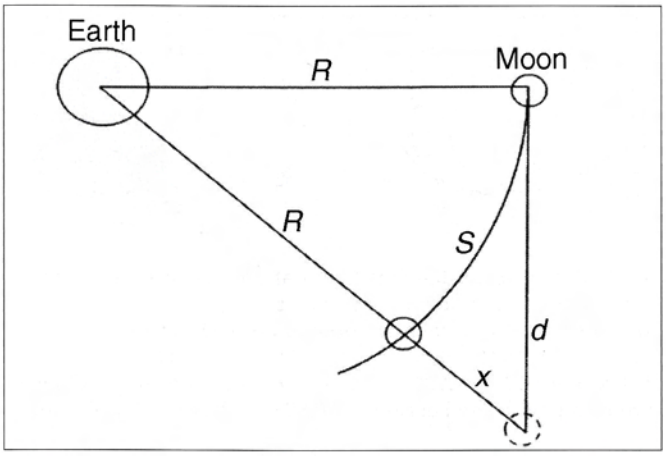

Fig. 1. The motion of the Moon if the Earth were to suddenly vanish. Distance *d* and arch length *s* approach one another in the limit of short time limits.

the acceleration a with which the Moon continuously falls toward the Earth, missing it all the time." Additionally, this acceleration must be along the same line as distance x since this is the direction of the net force on the Moon, thus revealing the centripetal nature of the vector.

Gamow then calculates the centripetal acceleration of the Moon and shows it is approximately 3640 times smaller than g, which is the acceleration of the falling apple. "Thus it became clear that the force of gravity decreases with the distance from the Earth, but what is the law governing this decrease?" Knowing the Moon is approximately 60.1 times further from the Earth's center than the apple, Newton realizes the force of attraction must decrease as the inverse square of the distance.

References

1. George Gamow, *Gravity* (Anchor Books, New York, 1962), pp. 37–46.
2. Ernest Zebrowski Jr., "On the derivation of the centripetal acceleration formula," *Phys. Teach.* **10**, 527–528 (Dec. 1972).
3. Bill Wedemeyer, "Centripetal acceleration–A simpler derivation," *Phys. Teach.* **31**, 238–239 (April 1993).

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